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Extinction theorem and propagation of electromagnetic waves between two anisotropic materials

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Abstract. We systematically investigate the reflection and refraction of an electromagnetic wave between two semi-infinite anisotropic magnetoelectric materials. Using the integral formulation of Hertz vectors and the principle of superposition, we generalize the extinction theorem and derive the propagation characteristics of wave. Applying the results obtained, we find a general origin of Brewster effect. We also show that, through choosing appropriate material parameters, oblique or omnidirectional total transmission can occur to TE and TM waves. Compared to the traditional method, the method used here discloses the underlying mechanism of wave propagation between two arbitrary anisotropic materials and can be applied to other problems of propagation.

PACS. 41.20.Jb Electromagnetic wave propagation; radiowave propagation – 42.25.Fx Diffraction and scattering – 78.20.Ci Optical constants (including refractive index, complex dielectric constant, absorption, reflection and transmission coefficients, emissivity)

1 Introduction

In classical electromagnetism there are two well-known approaches to the propagation of electromagnetic waves. The first is to solve Maxwell's equations with boundary conditions and the second is to use integral equation treatment with Ewald-Oseen extinction theorem [1]. Compared to the former used traditionally, the latter can give much deeper physical insights into the interaction of electromagnetic wave with material [1–5]. The integral equation treatment plays a key role in light scattering theory [6,7]. Using the extinction theorem, the propagations of electromagnetic waves through a semi-infinite isotropic material [3,8] and an isotropic slab [9] have been studied. Recently the Brewster mechanism is explained for light incident on an isotropic material with negative index [10].

In the previous works using the integral equation treatment, most deal with the propagation of electromagnetic waves incident from free space into isotropic materials. However, the opposite situation from an material into free space and especially the situation between two materials is rarely studied. Then, whether the methods used in the previous work are applicable to the two situations? To answer this question, it is necessary to generalize the extinction theorem to the propagation between two materials. On the other hand, a recent advent of artificial materials, named as negative-refraction materials, arouses much interest in the field of optics [11–26]. Since the negative-refraction materials are actually anisotropic magnetoelectric, it is also necessary to extend the extinction theorem from isotropic materials to anisotropic materials.

The purpose of this paper is to generalize the integral equation treatment to the propagation of electromagnetic wave between two anisotropic magnetoelectric materials. Using the integral formulation of Hertz vectors and the principle of superposition, we derive the properties of propagation and generalize the extinction theorem, so that the propagation between two arbitrary materials can be investigated in a unified framework. We also give a general explanation for the mechanism of Brewster effect. We show that, through choosing appropriate material parameters, oblique or omnidirectional total transmission [26–28] can occur to TE and TM waves. The methods used do not require boundary conditions, but can reveal the interaction of light with materials, and avoid the difficulties in integrations and complex calculations usually encountered in using extinction theorem [3,6]. So the methods can be applied to other problems of wave propagation in materials, such as scattering of light.

2 Reflection, refraction, and extinction theorem

In this section, we first employ the formulation of Hertz vector and the principle of superposition to deduce the radiated fields generated by dipoles in the propagation of

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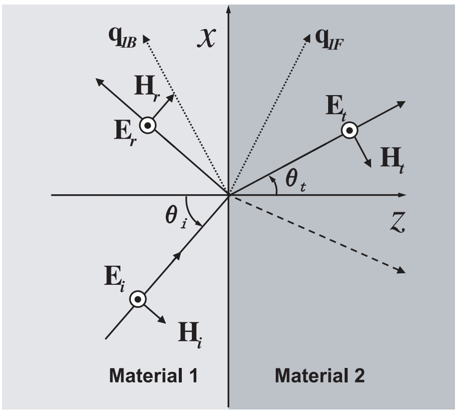

Fig. 1. Schematic diagram for the reflection and refraction of TE waves at the interface between vacuum and an anisotropic material. $\mathbf{q}_{1F}$ and $\mathbf{q}_{1B}$ are the forward and backward vacuum wave vectors generated by the two materials, respectively. Both of them should be canceled out, which correspond to the two expressions of the extinction theorem. The dashed line denotes the possible ray of negative refraction.

waves between two anisotropic dielectric-magnetic materials. Then we derive the real reflected and transmitted fields and the Fresnel's coefficients. At the same time, we generalize the Ewald-Oseen extinction theorem. Throughout the paper SI units are used.

Let us consider a monochromatic electromagnetic field of $\mathbf{E}_i = \mathbf{E}_{i0}\exp(i\mathbf{k}_i\cdot\mathbf{r}-i\omega t)$ and $\mathbf{H}_i = \mathbf{H}_{i0}\exp(i\mathbf{k}_i\cdot\mathbf{r}-i\omega t)$ incident from an anisotropic material into another one filling the semi-infinite space $z>0$ with $\mathbf{k}_i = k_{ix}\hat{\mathbf{x}} - k_{iz}\hat{\mathbf{z}}$. The schematic diagram is in Figure 1. Since the material responds linearly, all the fields have the same dependence of $\exp(-i\omega t)$ which will be omitted subsequently. Assume the reflected fields are $\mathbf{E}_r = \mathbf{E}_{r0}\exp(i\mathbf{k}_r\cdot\mathbf{r})$ and $\mathbf{H}_r = \mathbf{H}_{r0}\exp(i\mathbf{k}_r\cdot\mathbf{r})$, and the transmitted fields are $\mathbf{E}_t = \mathbf{E}_{t0}\exp(i\mathbf{k}_t\cdot\mathbf{r})$ and $\mathbf{H}_t = \mathbf{H}_{t0}\exp(i\mathbf{k}_t\cdot\mathbf{r})$. For simplicity, the permittivity and permeability tensors of materials are assumed diagonal simultaneously in the principal coordinate system, $\boldsymbol{\varepsilon}_j = \mathrm{diag}[\varepsilon_{jx},\varepsilon_{jy},\varepsilon_{jz}]$, $\boldsymbol{\mu}_j = \mathrm{diag}[\mu_{jx},\mu_{jy},\mu_{jz}]$, $j=1,2$.

Following the molecular optics theory, a bulk material can be regarded as a collection of molecules (or atoms) embedded in the vacuum. Driven by the external field, the molecules are brought into oscillations and then secondary waves are generated by the induced dipoles. The radiated electric field by dipoles is decided by [1]

$$\mathbf{E}_{rad} = \nabla(\nabla\cdot\mathbf{\Pi}_e) - \varepsilon_0\mu_0\frac{\partial^2\mathbf{\Pi}_e}{\partial t^2} - \mu_0\nabla\times\frac{\partial\mathbf{\Pi}_m}{\partial t} \tag{1}$$

and the generated magnetic field is

$$\mathbf{H}_{rad} = \nabla(\nabla\cdot\mathbf{\Pi}_m) - \varepsilon_0\mu_0\frac{\partial^2\mathbf{\Pi}_m}{\partial t^2} + \varepsilon_0\nabla\times\frac{\partial\mathbf{\Pi}_e}{\partial t}. \tag{2}$$

Here $\mathbf{\Pi}_e$ and $\mathbf{\Pi}_m$ are the Hertz vectors,

$$\mathbf{\Pi}_e(\mathbf{r}) = \int\frac{\mathbf{P}(\mathbf{r}')}{\varepsilon_0}G(\mathbf{r}-\mathbf{r}')\mathrm{d}\mathbf{r}', \tag{3}$$

$$\mathbf{\Pi}_m(\mathbf{r}) = \int\mathbf{M}(\mathbf{r}')G(\mathbf{r}-\mathbf{r}')\mathrm{d}\mathbf{r}'. \tag{4}$$

The dipole moment density of electric dipoles $\mathbf{P}$ and that of magnetic dipoles $\mathbf{M}$ are related to the associated field as $\mathbf{P} = \varepsilon_0\boldsymbol{\chi}_e\cdot\mathbf{E}$, $\mathbf{M} = \boldsymbol{\chi}_m\cdot\mathbf{H}$, where the electric susceptibility $\boldsymbol{\chi}_e = (\boldsymbol{\varepsilon}/\varepsilon_0)-1$ and the magnetic susceptibility $\boldsymbol{\chi}_m = (\boldsymbol{\mu}/\mu_0)-1$. The Green function is $G(\mathbf{r}-\mathbf{r}') = \exp(ik_0|\mathbf{r}-\mathbf{r}'|)/(4\pi|\mathbf{r}-\mathbf{r}'|)$, where k_0 is the wave number in vacuum. In the first medium the dipoles produce forward waves as well as backward waves. So, we assume the associated dipole moment densities have the following forms

$$\begin{aligned}\mathbf{P}_1 &= \mathbf{P}_{1F}\exp(i\mathbf{k}_{1F}\cdot\mathbf{r}) + \mathbf{P}_{1B}\exp(-i\mathbf{k}_{1B}\cdot\mathbf{r}),\\ \mathbf{M}_1 &= \mathbf{M}_{1F}\exp(i\mathbf{k}_{1F}\cdot\mathbf{r}) + \mathbf{M}_{1B}\exp(-i\mathbf{k}_{1B}\cdot\mathbf{r}),\end{aligned} \tag{5}$$

while $\mathbf{P}_2 = \mathbf{P}_{2F}\exp(i\mathbf{k}_{2F}\cdot\mathbf{r})$ and $\mathbf{M}_2 = \mathbf{M}_{2F}\exp(i\mathbf{k}_{2F}\cdot\mathbf{r})$ in the second material, where F and B label the forward and backward propagating waves, respectively. To determine the Hertz vectors we firstly represent the Green function in the Fourier form [8]. Then, inserting it into equation (3) and using the delta function definition and contour integration method, the Hertz vectors can be evaluated.

Following the principle of superposition, the fields in the right region produced by the dipoles of the two media add up to the transmitted field. Then, we have

$$\mathbf{E}_t = \mathbf{E}_{rad}^{1.right} + \mathbf{E}_{rad}^2, \tag{6}$$

where the contribution from the first medium to the right side, $\mathbf{E}_{rad}^{1.right}$, can be calculated as

$$\mathbf{E}_{rad}^{1.right} = -\frac{\mathbf{Q}(\mathbf{q}_{1F},\boldsymbol{P}_{1F})}{2q_{1z}(q_{1z}-k_{1Fz})} - \frac{\mathbf{Q}(-\mathbf{b}_{1B},\boldsymbol{P}_{1B})}{2b_{1z}(b_{1z}+k_{1Fz})}, \tag{7}$$

the field radiated by the second medium itself, $\mathbf{E}_{rad}^2$, can be obtained

$$\mathbf{E}_{rad}^2 = \frac{\mathbf{Q}(\mathbf{k}_{2F},\boldsymbol{P}_{2F})}{k_{2F}^2-q_{2F}^2} + \frac{\mathbf{Q}(\mathbf{q}_{2F},\boldsymbol{P}_{2F})}{2q_{2z}(q_{2z}-k_{2Fz})}, \tag{8}$$

and $\mathbf{Q}$ is an auxiliary function

$$\begin{aligned}\mathbf{Q}(\mathbf{K},\boldsymbol{P}) \equiv -\frac{1}{\varepsilon_0}\Big[&\mathbf{K}\times\mathbf{K}\times\boldsymbol{P} + (\mathbf{K}^2-\varepsilon_0\mu_0\omega^2)\boldsymbol{P}\\ &+\omega\mu_0\varepsilon_0\mathbf{K}\times\boldsymbol{M}\Big]\exp(i\mathbf{K}\cdot\mathbf{r})\end{aligned} \tag{9}$$

where $\mathbf{M} = \boldsymbol{\chi}_m\cdot\{\mathbf{k}\times[\mathbf{P}/(\varepsilon_0\boldsymbol{\chi}_e)]/\boldsymbol{\mu}\}/\omega$, $\mathbf{q}_{jF} = k_{jFx}\hat{\mathbf{x}} + q_{jz}\hat{\mathbf{z}}$, $\mathbf{q}_{1B} = k_{1Fx}\hat{\mathbf{x}} - q_{1z}\hat{\mathbf{z}}$, $q_{jz}^2 = k_0^2 - k_{jFx}^2$, $\mathbf{b}_{1F} = k_{1Bx}\hat{\mathbf{x}} + b_{1z}\hat{\mathbf{z}}$, $\mathbf{b}_{1B} = k_{1Bx}\hat{\mathbf{x}} - b_{1z}\hat{\mathbf{z}}$, $b_{1z}^2 = k_0^2 - k_{1Bx}^2$, $j=1,2$. Substituting equations (7) and (8) into (6), and considering the transmitted field with the form

$$\mathbf{E}_t = \frac{\mathbf{P}_{2F}}{\varepsilon_0\boldsymbol{\chi}_{2e}}\exp(i\mathbf{k}_{2F}\cdot\mathbf{r}), \tag{10}$$

we come to the following conclusions by a self-consistent analysis. (i) From terms with the phase factor $\exp(i\mathbf{k}_{2F}\cdot\mathbf{r})$ in equation (6) yield $\mathbf{k}_{2F}=\mathbf{k}_t$ and the dispersion relation

$$\frac{k_{tx}^2}{\mu_{2z}\varepsilon_{2y}}+\frac{k_{tz}^2}{\mu_{2x}\varepsilon_{2y}}=\omega^2, \qquad \frac{k_{tx}^2}{\varepsilon_{2z}\mu_{2y}}+\frac{k_{tz}^2}{\varepsilon_{2x}\mu_{2y}}=\omega^2 \quad (11)$$

for TE and TM waves, respectively. (ii) We know that $\mathbf{q}_{1F}$, $\mathbf{q}_{2F}$ and $\mathbf{b}_{1B}$ are all vacuum wave vectors. Since only $\mathbf{k}_{2F}$ appears in the final transmitted field, they all should be extinguished. So we conclude that $q_{1z}=q_{2z}=b_{1z}$ and $k_{1Fx}=k_{2Fx}=-k_{1Bx}$. Then, comes naturally the Snell's law: $k_{1F}\sin\theta_i=k_{2F}\sin\theta_t$. At the same time,

$$\frac{\mathbf{Q}(\mathbf{q}_{1F},\boldsymbol{P}_{2F})}{q_{1z}-k_{2Fz}}-\frac{\mathbf{Q}(\mathbf{q}_{1F},\boldsymbol{P}_{1F})}{q_{1z}-k_{1Fz}}-\frac{\mathbf{Q}(\mathbf{q}_{1F},\boldsymbol{P}_{1B})}{q_{1z}+k_{1Fz}}=0. \quad (12)$$

Equation (12) is the generalized expression of *the extinction theorem about the forward vacuum waves.* It describes how the radiation field produced by the dipoles of the first medium is extinguished by the counterpart in the second medium.

Now we study the fields in the first material. Similarly, all the fields radiated by the whole space are superposed to form the incident and reflected field

$$\mathbf{E}_i+\mathbf{E}_r=\mathbf{E}_{rad}^1+\mathbf{E}_{rad}^{2.left}, \quad (13)$$

where the field radiated by the first medium, $\mathbf{E}_{rad}^1$, can be calculated out

$$\mathbf{E}_{rad}^1=\frac{\mathbf{Q}(\mathbf{k}_{1F},\boldsymbol{P}_{1F})}{k_{1F}^2-q_{1F}^2}+\frac{\mathbf{Q}(\mathbf{q}_{1B},\boldsymbol{P}_{1F})}{2q_{1z}(q_{1z}+k_{1Fz})} +\frac{\mathbf{Q}(-\mathbf{k}_{1B},\boldsymbol{P}_{1B})}{k_{1B}^2-b_{1B}^2}+\frac{\mathbf{Q}(\mathbf{q}_{1B},\boldsymbol{P}_{1B})}{2q_{1z}(q_{1z}-k_{1Bz})}, \quad (14)$$

the contribution $\mathbf{E}_{rad}^{2.left}$ from the second medium to the left half-space can be obtained as

$$\mathbf{E}_{rad}^{2.left}=-\frac{\mathbf{Q}(\mathbf{q}_{2B},\boldsymbol{P}_{2F})}{2q_{2z}(q_{2z}+k_{2Fz})}. \quad (15)$$

The incident and reflected fields may be written as

$$\mathbf{E}_i=\frac{\mathbf{P}_{1F}}{\varepsilon_0\boldsymbol{\chi}_{1e}}\exp(i\mathbf{k}_{1F}\cdot\mathbf{r}), \quad \mathbf{E}_r=\frac{\mathbf{P}_{1B}}{\varepsilon_0\boldsymbol{\chi}_{1e}}\exp(-i\mathbf{k}_{1B}\cdot\mathbf{r}), \quad (16)$$

respectively. Inserting equations (14), (15) and (16) into (13) which must hold true for everywhere in the left half-space, we come to the following conclusions. (i) From terms with the phase factor $\exp(i\mathbf{k}_{1F}\cdot\mathbf{r})$ or $\exp(i\mathbf{k}_{1B}\cdot\mathbf{r})$ in equation (13) follow that $\mathbf{k}_{1F}=\mathbf{k}_i$, $\mathbf{k}_{1B}=\mathbf{k}_r$ and the dispersion relation is like equation (11) after replacing the subscripts 2 with 1 and t with i, respectively. (ii) $\mathbf{q}_{1B}$ and $\mathbf{q}_{2B}$ are vacuum wave vectors and should be extinguished. So, we have $\mathbf{q}_{1B}=\mathbf{q}_{2B}$ and

$$\frac{\mathbf{Q}(\mathbf{q}_{1B},\boldsymbol{P}_{1F})}{q_{1z}+k_{1Fz}}+\frac{\mathbf{Q}(\mathbf{q}_{1B},\boldsymbol{P}_{1B})}{q_{1z}-k_{1Fz}}-\frac{\mathbf{Q}(\mathbf{q}_{1B},\boldsymbol{P}_{2F})}{q_{1z}+k_{2Fz}}=0, \quad (17)$$

which is a new expression of *the extinction theorem* we find. It shows how *the backward vacuum field* produced by the dipoles of the second medium is extinguished by that in the first medium.

Solving the set of equations (10), (12), (16), and (17), we obtain the reflection coefficient $R_E=E_{r0}/E_{i0}$ and the transmission coefficient $T_E=E_{t0}/E_{i0}$ for TE waves

$$R_E=\frac{\mu_{2x}k_{iz}-\mu_{1x}k_{tz}}{\mu_{2x}k_{iz}+\mu_{1x}k_{tz}}, \quad T_E=\frac{2\mu_{2x}k_{iz}}{\mu_{2x}k_{iz}+\mu_{1x}k_{tz}}. \quad (18)$$

Analogously, we can discuss TM waves and obtain similar results after replacing $\boldsymbol{\mu}$ with $\boldsymbol{\varepsilon}$, considering the expressions of $\mathbf{E}$ in equation (1) and $\mathbf{H}$ in equation (2).

Hence, we have obtained the generalized extinction theorem, i.e. equations (12) and (17). Then, the propagation between two arbitrary materials can be studied in a unified framework: under the action of external field in the first material, the molecules in the two materials are driven to oscillate and generate induced fields. The transmitted wave is the result of superposition of the induced vacuum field from the first medium and the fields radiated by the induced dipoles in the second medium. The reflected wave is the sum of the backward vacuum field from the second medium and the backward fields induced by the dipoles in the first medium. Note that equations (12) and (17) should hold true for every point in the associated half-spaces, which indicates that the cancellation of vacuum waves occurs everywhere inside the materials.

3 Origin of Brewster effect

In what follows we apply the above conclusions to discussing the origin of Brewster effect.

If the power reflectivity $|R|^2=0$, there is no reflected wave and the incident angle is called Brewster angle [27,29]. From equations (17) follows the reflected field magnitude

$$\mathbf{E}_{r0}=\frac{1}{q_{1z}+k_{iz}\mu_0/\mu_{1x}}\left\{\frac{\mathbf{q}_{1B}\times[\mathbf{q}_{1B}\times(\boldsymbol{\chi}_{e1}\cdot\mathbf{E}_{i0})]}{q_{1z}+k_{iz}} +\frac{\mathbf{q}_{1B}\times\{\boldsymbol{\chi}_{m1}\cdot[\mu_0\boldsymbol{\mu}_1^{-1}\cdot(\mathbf{k}_i\times\mathbf{E}_{i0})]\}}{q_{1z}+k_{iz}} +\frac{\mathbf{q}_{1B}\times[\mathbf{q}_{1B}\times(\boldsymbol{\chi}_{e2}\cdot\mathbf{E}_{t0})]}{q_{1z}+k_{tz}} +\frac{\mathbf{q}_{1B}\times\{\boldsymbol{\chi}_{m2}\cdot[\mu_0\boldsymbol{\mu}_2^{-1}\cdot(\mathbf{k}_t\times\mathbf{E}_{t0})]\}}{q_{1z}+k_{tz}}\right\}. \quad (19)$$

In the large bracket of equation (19), the first two terms denote the contributions (labeled as $\mathbf{E}_{r0}^i$) from electric and magnetic dipoles in the first medium to the reflected field, respectively, while the last two are the contributions (labeled as $\mathbf{E}_{r0}^t$) from the second medium. In order for zero reflection, it requires that $\mathbf{E}_{r0}^i+\mathbf{E}_{r0}^t=0$, from which follows the condition for Brewster effect: if the material parameters satisfy

$$\frac{\mu_{2z}\varepsilon_{2y}-\mu_{1z}\varepsilon_{1y}}{\mu_{1z}\mu_{2z}(\mu_{2x}\varepsilon_{1y}-\mu_{1x}\varepsilon_{2y})}>0, \quad (20)$$

Table 1. Existence conditions of Brewster angles for TE waves at the interface of two anisotropic media. Note: $*$ denotes either of $\pm$; $\forall$ indicates that Brewster angle exists for any parameter values.

μ_{1x}	μ_{1z}	ε_{1y}	μ_{2x}	μ_{2z}	ε_{2y}	Existence conditions
+	+	+	+	+	+	$\frac{\mu_{2x}}{\mu_{1x}} > \frac{\varepsilon_{2y}}{\varepsilon_{1y}} \cap \frac{\varepsilon_{2y}}{\varepsilon_{1y}} > \frac{\mu_{1z}}{\mu_{2z}}, \frac{\mu_{2x}}{\mu_{1x}} < \frac{\varepsilon_{2y}}{\varepsilon_{1y}} \cap \frac{\varepsilon_{2y}}{\varepsilon_{1y}} < \frac{\mu_{1z}}{\mu_{2z}}$
+	+	+	−	+	+	$\frac{\varepsilon_{2y}}{\varepsilon_{1y}} < \frac{\mu_{1z}}{\mu_{2z}}$
+	+	+	+	−	+	$\frac{\mu_{2x}}{\mu_{1x}} > \frac{\varepsilon_{2y}}{\varepsilon_{1y}}$
$*$	$*$	$*$	+	+	−	$\times$
+	+	−	$*$	$*$	$*$	$\times$
+	+	+	+	−	−	$\frac{\varepsilon_{2y}}{\varepsilon_{1y}} > \frac{\mu_{1z}}{\mu_{2z}}$
+	+	+	−	+	−	$\frac{\mu_{2x}}{\mu_{1x}} < \frac{\varepsilon_{2y}}{\varepsilon_{1y}}$
+	−	+	+	−	+	$\frac{\mu_{2x}}{\mu_{1x}} < \frac{\varepsilon_{2y}}{\varepsilon_{1y}} \cap \frac{\varepsilon_{2y}}{\varepsilon_{1y}} > \frac{\mu_{1z}}{\mu_{2z}}, \frac{\mu_{2x}}{\mu_{1x}} > \frac{\varepsilon_{2y}}{\varepsilon_{1y}} \cap \frac{\varepsilon_{2y}}{\varepsilon_{1y}} < \frac{\mu_{1z}}{\mu_{2z}}$
+	−	+	−	+	+	$\forall$
−	+	+	−	+	+	$\frac{\mu_{2x}}{\mu_{1x}} < \frac{\varepsilon_{2y}}{\varepsilon_{1y}} \cap \frac{\varepsilon_{2y}}{\varepsilon_{1y}} > \frac{\mu_{1z}}{\mu_{2z}}, \frac{\mu_{2x}}{\mu_{1x}} > \frac{\varepsilon_{2y}}{\varepsilon_{1y}} \cap \frac{\varepsilon_{2y}}{\varepsilon_{1y}} < \frac{\mu_{1z}}{\mu_{2z}}$
+	+	+	−	−	−	$\frac{\mu_{2x}}{\mu_{1x}} > \frac{\varepsilon_{2y}}{\varepsilon_{1y}} \cap \frac{\varepsilon_{2y}}{\varepsilon_{1y}} > \frac{\mu_{1z}}{\mu_{2z}}, \frac{\mu_{2x}}{\mu_{1x}} < \frac{\varepsilon_{2y}}{\varepsilon_{1y}} \cap \frac{\varepsilon_{2y}}{\varepsilon_{1y}} < \frac{\mu_{1z}}{\mu_{2z}}$
+	−	+	+	−	−	$\forall$
+	−	+	−	+	−	$\frac{\mu_{2x}}{\mu_{1x}} < \frac{\varepsilon_{2y}}{\varepsilon_{1y}} \cap \frac{\varepsilon_{2y}}{\varepsilon_{1y}} > \frac{\mu_{1z}}{\mu_{2z}}, \frac{\mu_{2x}}{\mu_{1x}} > \frac{\varepsilon_{2y}}{\varepsilon_{1y}} \cap \frac{\varepsilon_{2y}}{\varepsilon_{1y}} < \frac{\mu_{1z}}{\mu_{2z}}$
+	−	−	−	+	+	$\frac{\mu_{2x}}{\mu_{1x}} < \frac{\varepsilon_{2y}}{\varepsilon_{1y}} \cap \frac{\varepsilon_{2y}}{\varepsilon_{1y}} > \frac{\mu_{1z}}{\mu_{2z}}, \frac{\mu_{2x}}{\mu_{1x}} > \frac{\varepsilon_{2y}}{\varepsilon_{1y}} \cap \frac{\varepsilon_{2y}}{\varepsilon_{1y}} < \frac{\mu_{1z}}{\mu_{2z}}$

Brewster effect will occur at the incident angle

$$\theta_B^{TE} = \cot^{-1}\sqrt{\frac{\mu_{1x}^2(\mu_{2z}\varepsilon_{2y} - \mu_{1z}\varepsilon_{1y})}{\mu_{1z}\mu_{2z}(\mu_{2x}\varepsilon_{1y} - \mu_{1x}\varepsilon_{2y})}}. \tag{21}$$

If $\mu_{2x}/\mu_{1x} = \varepsilon_{2y}/\varepsilon_{1y} \cap \varepsilon_{2y}/\varepsilon_{1y} \neq \mu_{1z}/\mu_{2z}$, then $\theta_B^{TE} = 0$; When $\mu_{2x}/\mu_{1x} \neq \varepsilon_{2y}/\varepsilon_{1y} \cap \varepsilon_{2y}/\varepsilon_{1y} = \mu_{1z}/\mu_{2z}$, $\theta_B^{TE} = \pi/2$; If $\mu_{2x}/\mu_{1x} = \varepsilon_{2y}/\varepsilon_{1y} \cap \varepsilon_{2y}/\varepsilon_{1y} = \mu_{1z}/\mu_{2z}$, then Brewster effect will occur for any angle of incidence, which may lead to important applications in optics. All the typical and nontrivial sign combinations of $\boldsymbol{\varepsilon}_j$ and $\boldsymbol{\mu}_j$ are shown in Table 1. Note that, if the first medium is vacuum, $\mathbf{E}_{r0}^i = 0$, then zero reflection can occur only when $\mathbf{E}_{r0}^t = 0$. In other words, the Brewster effect is because the contributions from electric and magnetic dipoles of the medium to the reflected field in vacuum add up to zero, which is in accordance with the conclusion in reference [10].

To illustrate the above conclusions, we calculate three examples of wave propagations presented in Figure 2. The power reflectivities of the three cases, which can be obtained by the conventional Maxwell approach, are plotted in (a), and the corresponding field magnitudes, which are obtained by the method in the present paper, are plotted in (b), (c), and (d), respectively. Because the first medium is vacuum in (b), the reflected field is formed by the radiated electric fields ($\mathbf{E}_{r0}^t$) generated by the second mediums, then Brewster angle appears when $\mathbf{E}_{r0}^t = 0$. In (c), oblique total transmission appears [26,27] when $\mathbf{E}_{r0}^i + \mathbf{E}_{r0}^t = 0$ and the incident angle is Brewster angle $\theta_{B(c)}^{TE}$. In (d), $\mathbf{E}_{r0}^i + \mathbf{E}_{r0}^t \equiv 0$, so omnidirectional total transmission [28] occurs. Comparing the results in (a) and those in (b), (c), and (d), one can see that the results by using the method in the present paper are in agreement with those by the traditional approach. At the same time, the former, i.e., (b), (c), and (d), can provide more microscopic view of point on the mechanism of Brewster angle.

The previous results are based on the assumption of plane wave. Since all physical sources of electromagnetic waves produce radiation fields of finite spatial and temporal extent, it is more essential to consider incident waves as localized wave packets. Therefore, we also follow the method in reference [18] to simulate in Figure 3 a beam $\mathbf{E}_i = \mathbf{E}_{i0}\int \mathrm{d}k_\perp e^{i(\mathbf{k}_0+\mathbf{k}_\perp)\cdot\mathbf{r}} f(k_\perp)$ incident from an anisotropic material to another where $f(k_\perp)$ is the Gaussian modulation. In Figure 3, the two materials in (a)-(c) and (d)-(f) correspond to those in (c) and (d) of Figure 2, respectively. The oblique total negative refraction [26] appears in (b) and the omnidirectional total transmission

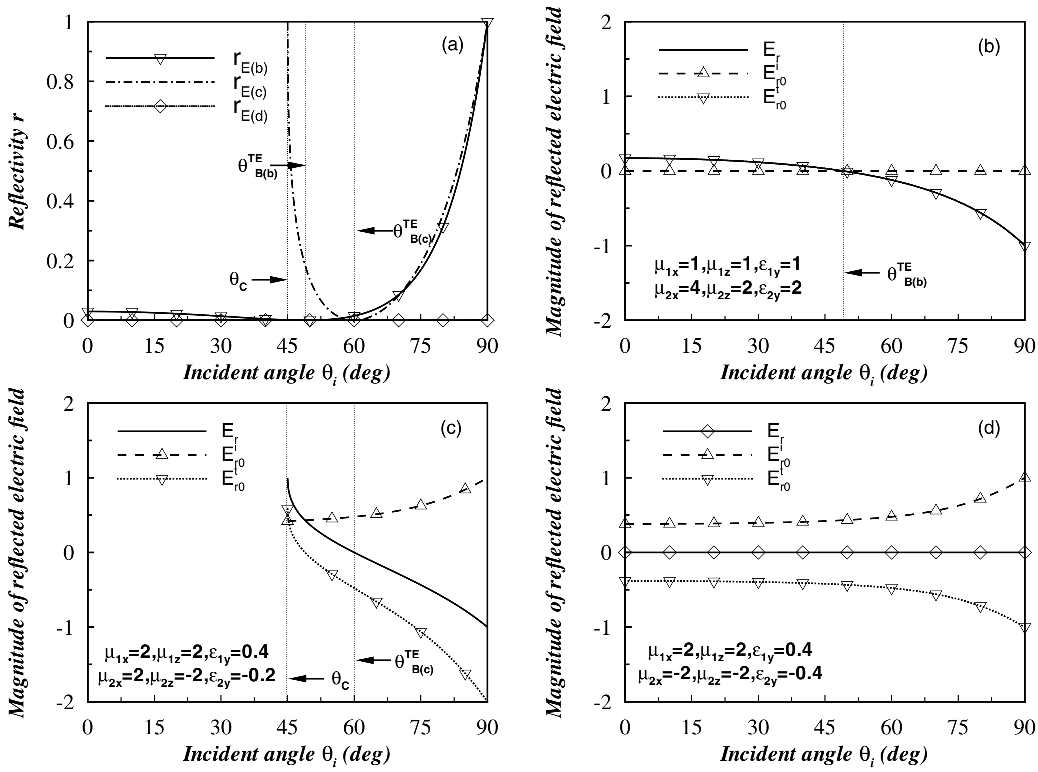

Fig. 2. Reflectivity and reflected field magnitudes, normalized by the corresponding incident field magnitudes, for TE wave incident incident on the interface between two anisotropic materials. The three curves of reflectivity in (a) correspond the case in (b), (c), and (d), respectively. Since the first medium is vacuum in (b), the radiated electric fields ($\mathbf{E}^t_{r0}$) generated by the second medium forms the reflected field, then Brewster angle appears when $\mathbf{E}^t_{r0} = 0$. In (c), oblique total transmission appears at $\theta^{TE}_{B(c)}$ and θ_c is the critical angle of incidence. In (d), $\mathbf{E}^i_{r0} + \mathbf{E}^t_{r0} \equiv 0$, so omnidirectional total transmission occurs.

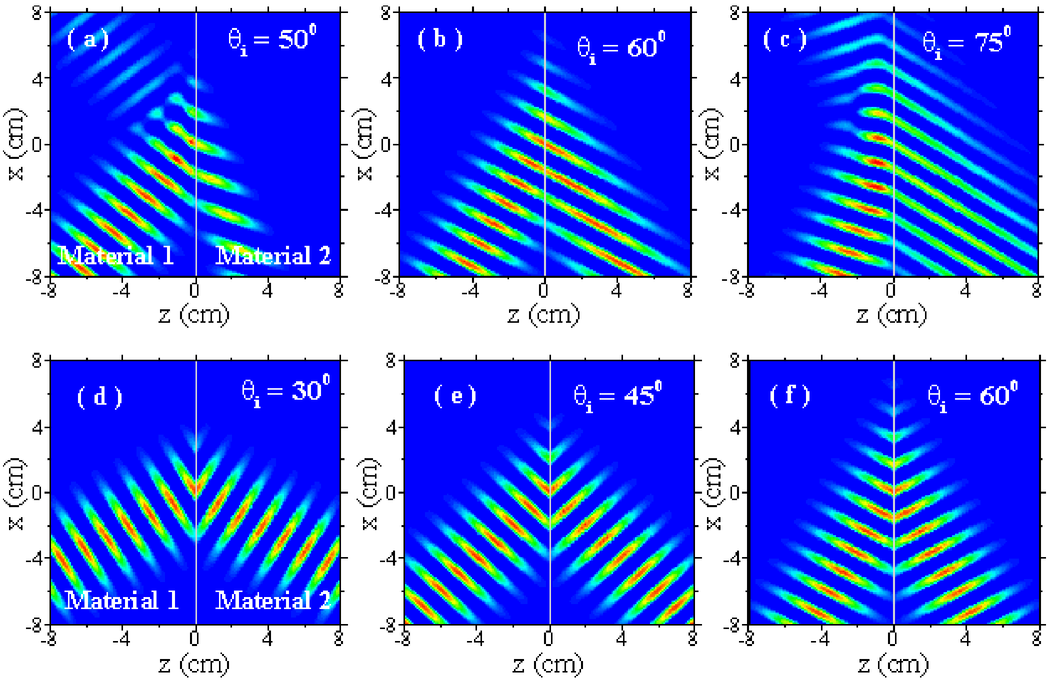

Fig. 3. (Color online) Reflection and refraction of a Gaussian beam incident on the interface between two anisotropic materials. For (a)-(c) and (d)-(f) the two materials correspond to those in (c) and (d) of Figure 2, respectively. The phase and the energy flow are refracted regularly and anomalously, respectively in (a)-(c), but are both refracted anomalously in (d)-(f). The oblique total negative refraction appears in (b) and the omnidirectional total transmission occurs to (d)-(f).

occurs to (d)-(f). The phase and the energy flow are refracted regularly and anomalously, respectively in (a)-(c), but are both refracted anomalously in (d)-(f). One can see that beam simulations are in agreement with theoretical analyses based on the assumption of plane wave. That indicates that our method and conclusion are convincing. Experimentally Brewster effect for TE waves has been realized with metamaterials [30]. So, one can realize zero reflection for TE and TM waves through choosing appropriate material parameters, which may be used to make polarizers or beam splitters.

4 Conclusion

In summary, we carried out a systematical investigation on the propagation of wave between two anisotropic magnetoelectric materials. Utilizing Hertz vectors and the principle of superposition, we derive the properties of propagation and generalize the extinction theorem, so that the propagation between two arbitrary materials can be investigated in a unified framework. We apply the results to explaining the physical origin of Brewster effect. We show that, through choosing appropriate material parameters, oblique or omnidirectional total transmission can occur. The methods do not require boundary conditions, but can disclose the process of light propagation in materials on a more fundamental level, and avoid complex calculations usually encountered in using the extinction theorem. So the methods can be applied to other problems of wave propagation, such as scattering of light [6,7].

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